

Stories from the Classrooms of Successful Mathematics Teachers: Painting a Picture of Effective Practice¹

Doug Clarke
Australian Catholic University
d.clarke@patrick.acu.edu.au

Barbara Clarke
Monash University
barbara.clarke@education.monash.edu.au

Most of us can recall teachers who made mathematics “come alive” during our time at school—teachers who challenged and yet supported us in our learning, teachers who seemed to know what we needed to learn and the best ways to enhance that learning. But what is it specifically that effective mathematics teachers do? In this article, we will describe a classroom research study of effective P-2 teachers that may provide some pieces of the puzzle. Effective teachers come in many shapes and sizes, but there may be important commonalities that can be shared, as well as differences. Our belief is that these findings give a framework that can be used as the basis for reflection on good practice, at preservice and inservice levels, and strong research evidence that recent developments in the teaching and learning of mathematics are moving in the right direction.

1.1 Background

For many years, researchers have sought to describe teacher behaviours that correlate positively with growth in student achievement. In the 1970s and 1980s, the so-called *process-product* research was conducted (see, e.g., Brophy & Good, 1986).

In these studies, the dependent variable was performance on standardised achievement tests. Most of these studies occurred in classrooms where the curriculum and classroom organisation were designed to support the teacher in *direct instruction*. Not surprisingly therefore, most of the effective teacher variables involved teacher behaviours associated with this style of teaching, including clarity of presentation, teacher wait time, and whole class questioning strategies.

Such research therefore is limited in what it offers of assistance to teachers seeking to operate in a classroom where meaning and understanding are the focus, although clarity of explanation and appropriate wait time are of course, still desirable. As will be discussed, an important difference between these studies and the present study was the quality of the assessment information that emerges from a one-to-one interview, particularly for young children.

Given the success of particular Asian countries in the Third International Mathematics and Science Study (TIMSS), there was considerable interest in describing commonalities between classrooms of particularly effective countries. Much interest focused on Japanese primary classrooms in particular. Through observation and videotape, Stigler and Stevenson (1991) conducted a study of 120 classrooms in Taipei (Taiwan), Sendai (Japan) and Minneapolis (USA). They characterised the classrooms in Japan and China as follows:

... coherent lessons that are presented in a thoughtful, relaxed and nonauthoritarian manner. Teachers frequently rely on students as sources of information. Lessons are oriented towards problem solving rather than rote mastery of facts and procedures and utilize many different types of representational materials. The role assumed by the teacher is that of knowledgeable guide, rather than that of prime dispenser of information and arbiter of what is correct. There is frequent verbal interaction in the classroom as the teacher attempts to stimulate students to produce, explain, and evaluate solutions to problems. (p. 14)

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It was not uncommon for these lessons to be organised around a single problem. Asian teachers tended to use short, frequent periods of seatwork, alternating between group discussion of problems and time for children to work through problems on their own. Reflection was a major feature of classrooms.

Brown (1998, p. 2) noted that international observational studies seem to show some agreement on some of the aspects of teacher quality which correlated with attainment. These included

- the use of higher order questions, statements and tasks which require thought rather than practice;
- emphasis on establishing, through dialogue, meanings and connections between different mathematical ideas and contexts;
- collaborative problem solving in class and small group settings; and
- more autonomy for students to develop and discuss their own methods and ideas.

Clarke (1997, p. 280), summarised research on the role of the teacher from studies in early years and middle school mathematics classrooms where problem solving was a major focus. He identified seven components of the role of the teacher that were common to these various settings. They were the following:

- The use of non-routine problems as the starting point and focus of instruction, without the provision of procedures for their solution.
- The adaptation of materials and instruction according to local contexts and the teacher's knowledge of students' interests and needs.
- The use of a variety of classroom organisational styles (individual, small group, whole class)
- The development of a "mathematical discourse community", with the teacher as "fellow player" who values and builds on students' solutions and methods.
- The identification and focus on the big ideas of mathematics.
- The use of informal assessment methods to inform instructional decisions.
- The facilitation of student reflection on activity and learning.

In a major study of effective primary school mathematics teaching in the United Kingdom, Askew, Brown, Rhodes, Johnson and Wiliam (1997) studied the practices of a number of different teachers, with varying levels of effectiveness. A specially designed oral mathematics test, "tiered" for different age ranges, was administered to the classes of 90 primary teachers, at the beginning and end of the school year. 2000 children were assessed. Considering relative learning gains, teachers were grouped as "highly effective," "effective," and "moderately effective." Data were then collected (using interviews, questionnaires and observations) on 18 case study teachers (six in each category), providing information on teachers' beliefs, pedagogical and mathematical subject knowledge, professional development experiences, and practices.

The teaching practices of the highly effective teachers

- connected different ideas of mathematics and different representations of each idea by means of a variety of words, symbols and diagrams;
- encouraged students to describe their methods and their reasoning, and used these descriptions as a way of developing understanding through establishing and emphasising connections;
- emphasised the importance of using whatever mental, written or electronic methods are most efficient for the problem at hand; and
- particularly emphasised the development of mental skills.

Possibly just as importantly, there were some characteristics of teaching practice that didn't enhance student learning.

Teachers who gave priority to pupils acquiring a collection of standard arithmetical methods over establishing understanding and connection produced lower numeracy gains. . . . Teachers who gave priority to the use of practical equipment rather than developing effective methods, and delayed the introduction of more abstract ideas until they felt a child was ready for them, also produced lower gains. (Brown, Askew, Rhodes, Johnson, & Wiliam, 1997, pp. 2-3).

Highly effective teachers of numeracy had knowledge and awareness of conceptual connections between the areas of the primary mathematics curriculum. There was no particular association between the amount of formal mathematics studied and student gains. Highly effective teachers were much more likely than other teachers to have undertaken mathematics-specific continuing professional development over an extended period.

Elsewhere, Brown (1999) noted that “quality teaching is more important than class organisation. . . . it’s not whether it’s whole-class, small group or individual teaching but rather what you teach and how you interact mathematically with children that seems to count” (p. 7).

Kieren (1997), in a reflection on the relationship between mathematics education research and practice, noted that because today, so much research occurs in classrooms, its potential direct relevance to practice is raised. He argued that research has pointed to alternative effective teaching practices and to new emphases:

- on listening to rather than simply listening for;
- on acting with students in doing mathematics rather than simply showing students how to do mathematics;
- on establishing effective discourses of mathematical argument or mathematical conversation rather than simply the discourse of telling, interrogating and evaluating;
- on the mechanisms of students’ mathematical thinking rather than simply on students’ answers;
- on the teacher and student as fully implicated by their actions each in the learning of the other; and
- on the teacher as co-developer of a lived mathematics curriculum not just the recipient of or a conduit for a pre-decided curriculum. (p. 33)

Kilpatrick and Silver (2000), in exploring challenges for mathematics education into the future, described how good mathematics teaching had been seen during different parts of the twentieth century:

Giving learners clear explanations, identifying clear instructional objectives, prefacing instruction on complex knowledge and skills with hierarchical sequences of purported prerequisites, breaking instruction into small steps learners can easily take on their own, immersing learners in dilemmas with which they must struggle, helping learners resolve one another’s confusions, tailoring instructional activities to individual learners’ perceived ways of learning. (p. 226)

The study discussed in this paper took place as part of the Early Numeracy Research Project (ENRP²) in Victoria, Australia. 354 P-2 teachers in 35 schools (approximately 240 per year) participated in a three-year research and professional development project, exploring the most effective approaches to the teaching of mathematics in the first three years of school. There were three key components of this project:

- a research-based framework of “growth points” in young children's mathematical learning (in Number, Measurement and Space);
- a 40-minute, one-on-one interview, used by all teachers with all children at the beginning and end of the school year;
- extensive professional development at central, regional and school levels, for all teachers, coordinators, and principals.

The framework of growth points will not be discussed in detail here (for a fuller description, see Clarke, 2001). However, the intention was to describe the typical “learning trajectory” of five- to eight-year olds.

There were four to six growth points in each mathematical domain. To illustrate the notion of a growth point, we will discuss the domain of Addition and Subtraction Strategies. Consider the child who is asked to find the total of two collections of objects. Many young children “count-all” to find the total, even when they are aware of the number of objects in each group. Other children realise that by starting at one of the numbers, they can count on to find the total. *Counting All* and *Counting On* are therefore two important growth points in children’s developing understanding of addition.

The six ENRP growth points for the domain of Addition and Subtraction Strategies are shown in Figure 1.

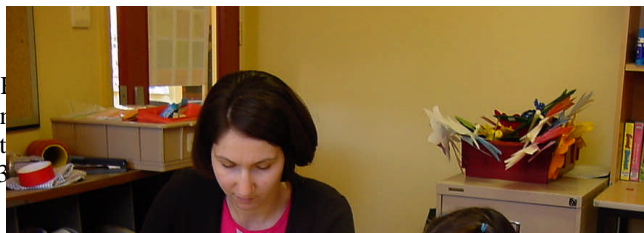
1. *Counts all to find the total of two collections.*
2. *Counts on from one number to find the total of two collections.*
3. *Given a subtraction situation, chooses appropriately from strategies including count-back, count-down-to and count-up-from.*
4. *Given an addition or subtraction problem, strategies such as doubles, commutativity, adding 10, tens facts, and other known facts are evident.*
5. *Given an addition or subtraction problem, strategies such as near doubles, adding 9, build to next ten, fact families and intuitive strategies are evident.*
6. *Given a range of tasks (including multi-digit numbers), can solve them mentally, using appropriate strategies and a clear understanding of key concepts.*

Figure 1. ENRP growth point descriptors for Addition and Subtraction Strategies.

A one-on-one, interactive, hands-on interview was then developed that could provide classroom teachers with rich information on what their children knew and could do (both individually and as a class) across a variety of domains, with particular insights into the strategies used in solving problems. The disadvantages of pen and paper tests have been well established by Clements (1995) and others, and these disadvantages are particularly evident with young children, where reading issues are of great significance. The interview has much to offer the teacher of young children, if time and resources permit.

To give a sense of the kinds of tasks

² The Early Numeracy Research Project (ENRP) was funded by Monash University, the Victorian Catholic Education Office (Melbourne), and the Victorian Government. It was funded in 35 project (“trial”) schools and 3



used in the interview, the following task enabled the teacher to determine whether a child, given a situation where they are required to find out “how many altogether” given two groups of known size, chose to count on, count all, used some other successful intuitive strategy, or were unable to solve the problem.

Words in italics are instructions to the interviewer; normal type are the words the interviewer uses with the child. “Teddies” refers to small, plastic teddy bears--a major feature of the interview.

18) Counting On

a) Please get four green teddies for me.

Place 9 green teddies on the table.

b) I have nine green teddies here (*show the child the nine teddies, and then screen the nine teddies with the ice-cream lid*).

That’s nine teddies hiding here and four teddies here (*point to the groups*).

c) Tell me how many teddies we have altogether... . Please explain how you worked it out.

d) (*If unsuccessful, remove the lid*). Please tell me how many there are altogether.

Figure 2. An excerpt from the Addition and Subtraction interview questions.

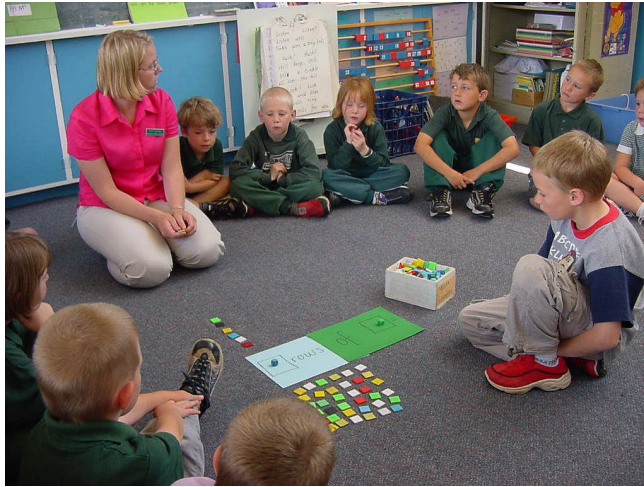
Although the full text of the ENRP interview involved around 60 tasks (with several sub-tasks in many cases), no child moved through all of these. The interview was of the form of a “choose your own adventure” story, in that the interviewer makes one of three decisions after each task, as instructed in the interview schedule. Given success with a task, the interviewer continued with the next task in the given mathematical domain as far as the child could go with success. Given difficulty with the task, the interviewer either abandoned that section of the interview and moved on to the next domain or moved into a detour, designed to elaborate more clearly the difficulty a child might be having with a particular content area.

At the time of writing the interview had been used with over 36,000 children at P-4. For each interview, teachers completed a four-page record sheet. The information on these sheets was then coded by a trained team of coders, assigning achieved growth points to each child for each domain. This process, including statistical measures to convert the growth point data to an interval scale, is discussed in detail in Rowley & Horne (2001).

The initial professional development was intended to prepare teachers to use the interview. Over the remaining three years of the project, the professional development focus was on taking what was learned from the interview to inform planning and teaching for maximum effectiveness, both cognitive and affective.

Identifying Particularly Effective Teachers

Given that the project occurred over three years, it was possible to use student interview data from the first two years to identify particularly effective teachers, for intensive study in the third year. In identifying effective teachers, our interest was in *growth* in student understanding across the school year. Some children, for reasons of home background, language background or other factors, come to school with less mathematical understanding and skills than others. Our emphasis on growth, however, allowed us to choose teachers that make a difference for all children.



By aggregating data on children's growth in terms of movement through the ENRP growth points, the first two years' data was used to identify particularly effective teachers—the ones whose children showed the greatest growth over two years, for each of two cohorts of children.

Using these data, we chose six teachers for case studies. There was one at Prep, one at Grade 1, one at Grade 2, and teachers of composite grades--a Grade P/1 and a Grade 1/2 teacher.

We also chose another Prep teacher who had made particularly impressive gains in a setting where almost all children were from non-English speaking backgrounds. The range of grades was in recognition that teaching Prep children mathematics is different in several ways to teaching Grade 2, for example.

Studying What Effective Teachers Do

The six case study teachers were studied intensively through use of the following data sources:

- five lesson observations by two researchers, incorporating detailed observer field notes, photographs of lessons and collection of artefacts (e.g., worksheets, student work samples, lesson plans);
- teacher interviews following the lessons;
- teacher questionnaires completed through the duration of the project; and
- teacher responses to other relevant questions and tasks posed to them.

Teachers were observed by two researchers, working together for five lessons. Lessons on three consecutive days were observed in the middle of the school year, and then lessons on two consecutive days were observed a couple of months later. Teachers were asked to focus on different broad content, of their choosing, in each of the two sets of lessons (e.g., Number the first set, Space the second). Both observers used laptop computers to take notes on the lesson, and teachers were interviewed after each lesson (interviews being audiotaped and transcribed), as they discussed their intentions for the lesson, and what transpired. Many photographs were taken also, to give a richer picture of the classroom environment and the nature of the activities used. In all, 86 researcher visits were made to schools during the case studies, including intensive practice of the whole team at one school.

Decisions needed to be made on the kinds of notes that would be taken on the lessons. The decision was taken to attempt to note as much of possible of what transpired in the lesson in a relatively “free” form. We were guided in this decision by the experience of others.

For example, Stigler and Baranes (1988) conducted mathematics classroom observations in three countries using two methods. In the first, a structured coding scheme was used with an elaborate time sampling plan. It was therefore possible to obtain estimates of the percentage time given to various classroom activities. In the second study, the researchers “decided to trade the greater reliability of an objective coding scheme, for the inherent richness of detailed narrative descriptions of mathematics lessons” (p. 294). The ENRP research team made the same decision.

We did however agree on a broad framework for the observations and interviews (see Figure 3).

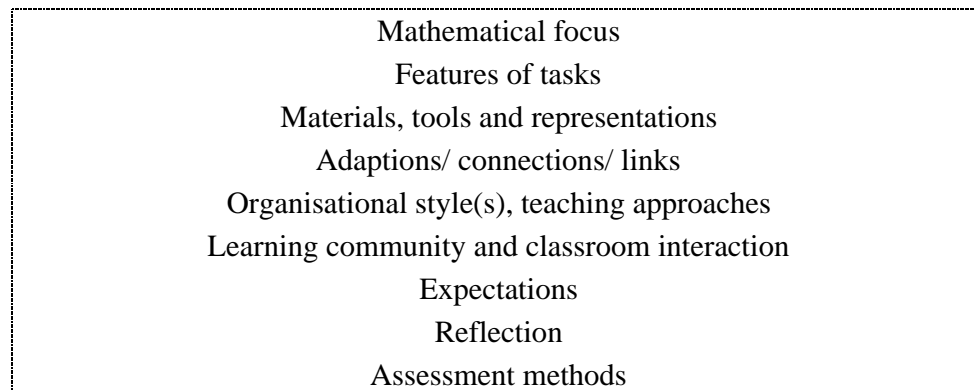


Figure 3. Categories within the ENRP lesson observation and analysis guide.

This framework was chosen to be quite broad, anticipating a criticism that the research team might have constrained their observations because of what they *hoped* they would see. We believe that most people with an interest in mathematics education, no matter their philosophical persuasion, would agree that these were reasonable components to study. Our aim was to describe the practice of demonstrably effective teachers and to ultimately look for common themes, not to judge.

At various stages during the case study, the team met together to describe to each other what they were seeing. “Critical friends”, not involved in the research, provided feedback on the kinds of themes they were hearing, as verbal reports were made. The first three lessons and the subsequent discussion prompted the team to focus on particular aspects in the last two lessons and interviews that had not necessarily been noted to that point, but which had emerged in team discussions.



What Can Be Said About Effective Teachers?

Following the lesson observations and interviews and a number of research team meetings, it was decided to use the original framework to describe the practices of effective teachers. It was agreed to list common elements where evidence was available for *at least four of the six teachers*. The description of effective teachers, as revealed in this study, is given in Figure 4.

<i>Effective teachers of P-2 mathematics . . .</i>	
Mathematical focus	<ul style="list-style-type: none"> • focus on important mathematical ideas • make the mathematical focus clear to the children
Features of tasks	<ul style="list-style-type: none"> • structure purposeful tasks that enable different possibilities, strategies and products to emerge • choose tasks that engage children and maintain involvement
Materials, tools and representations	<ul style="list-style-type: none"> • use a range of materials/representations/contexts for the same concept
Adaptions/ connections/ links	<ul style="list-style-type: none"> • use teachable moments as they occur • make connections to mathematical ideas from previous lessons or experiences
Organisational style(s), teaching approaches	<ul style="list-style-type: none"> • engage and focus children's mathematical thinking through an introductory, whole group activity • choose from a variety of individual and group structures and teacher roles within the major part of the lesson
Learning community and classroom interaction	<ul style="list-style-type: none"> • use a range of question types to probe and challenge children's thinking and reasoning • hold back from telling children everything • encourage children to explain their mathematical thinking/ideas • encourage children to listen and evaluate others' mathematical thinking/ideas, and help with methods and understanding • listen attentively to individual children • build on children's mathematical ideas and strategies
Expectations	<ul style="list-style-type: none"> • have high but realistic mathematical expectations of all children • promote and value effort, persistence and concentration
Reflection	<ul style="list-style-type: none"> • draw out key mathematical ideas during and/or towards the end of the lesson • after the lesson, reflect on children's responses and learning, together with activities and lesson content
Assessment methods	<ul style="list-style-type: none"> • collect data by observation and/or listening to children, taking notes as appropriate • use a variety of assessment methods • modifying planning as a result of assessment
Personal attributes of the teacher	<ul style="list-style-type: none"> • believe that mathematics learning can and should be enjoyable • are confident in their own knowledge of mathematics at the level they are teaching • show pride and pleasure in individuals' success

Figure 4. Common themes emerging from ENRP case studies of effective teachers.

The 25 common elements of good teaching sit well with the Teaching Principle of the U.S. National Council of Teachers of Mathematics (NCTM, 2000): "Effective mathematics teaching requires understanding what students know and need to learn and then

challenging and supporting them to learn it well” (p. 16).

What Does This Add to Our Knowledge About Effective Teachers?

As the reader considers this list, two points can be made:

- The six teachers were chosen from 150 who were involved in the first two years of the project on the basis of the growth in understanding and skills over two years of teaching, using an instrument in which we can have great confidence. The one-on-one interview provided far richer and more important data than can possibly be revealed by standardized testing, particularly in Grades P-2. That is, we can be confident that the teachers chosen as effective were done so on the basis of student improvement in the things we value.
- Although only six teachers were studied intensively, our visits to the classrooms of other project teachers (578 school visits in all), led us to believe that the features in Figure 4 were increasingly evident over the three years of the project in other classrooms, as teachers took what they had learned from the interviews about children’s mathematical thinking, and, working with colleagues, endeavoured to provide the kinds of activities and tasks that enhanced learning for all students. In a questionnaire at the end of the project, all teachers were asked to identify the greatest changes in their teaching practice. Among the most common were: more open-ended tasks and activities; more probing questioning/ asking why and how/ valuing children’s thinking; challenging and extending children/ higher expectations; more practical/ hands-on activities; and greater emphasis on reflection/ sharing.

Snapshots from the Classrooms of Effective P-2 Teachers

Sometimes, a list such as that in Figure 4 can seem a bit removed from the life and colour of the classroom. In order to give a picture of vibrant kinds of learning communities we observed during the study, we now discuss four classroom examples that illustrate the themes evident in the table.

Effective teachers structure purposeful tasks that enable different possibilities, strategies and products to emerge

The interviews made very clear the considerable range of knowledge and understanding within any classroom. As a result, teachers made extensive use of *open tasks*, encouraging children to share their strategies of solution. Examples of such tasks included:

- I rolled three dice and the total was 10. What might the dice have been?
- Two children measured the basketball court with rulers. Huong said it was 20 rulers. Yasmine said it was 19_. Why might that be?
- I drew a shape with four sides. What might my shape look like?
- I counted something in the room. There were exactly four. What might it have been?
- I bought some things at the supermarket and got 35 cents change. What did I buy and how much did each item cost?
- What can you find that is lighter than a potato but bigger than it?

Children accepted eagerly the challenge of these kinds of tasks, responding at their own level of understanding.

Effective teachers of mathematics focus on important mathematical ideas and make the mathematical focus clear to the children

Teachers noticed during the interview that although many children could read and write two- and three-digit numbers, quite a few had difficulty ordering one-digit numbers.

In school teams, they developed a range of games and activities that focused on this important mathematical idea.

One teacher asked children to cut up magazines and catalogs, taking out any numbers they could find. They then sorted these in out from smallest to largest. The two examples give a sense of range of children's responses.



Another teacher developed a card game, where the picture cards were removed from a standard pack, and children had half the pack each. At the same time, each child turns over a card, and the person with the larger of the two numbers takes both. Once again, there was a clear focus on ordering numbers from smaller to larger.



Effective teachers of mathematics encourage children to explain their mathematical thinking/ideas and build on children's mathematical ideas and strategies

One of the benefits of the regular use of the interview was that the kinds of tasks used and questions posed provided models of the kinds of tasks and questions that could be used in classrooms. Teachers commented that they found themselves using many more questions that probed children's thinking. Examples included:

- How did you work that out?
- Could you do that another way?
- How are these two objects the same, and how are they different?
- What happens if I change this here?
- What could you do next?
- Can you see a pattern in what you've found?
- Can you make up a new task using the same materials?

Effective early numeracy teachers use teachable moments as they occur

A teacher who taught a mixed Prep/Grade 1 class used children's literature regularly in her mathematics classes. One morning, she read *Counting on Frank* (Clement, 1994) to the children.



After she read the last page, she asked the children “who can remember how many jelly beans were in the jar?”

One of the children said that it was seven hundred and something and so the teacher said “I am going to read it again I want someone to write it on the white board.”

One of the children wrote 70045.

Ms Grade P/1 commented that the child had done a good job, but noted that this was how we read it, but we wrote it a different way. The next child wrote 7045, and a third wrote 745.

The teacher had some Montessori cards beside her chair and she showed the class the units and 10s.

She showed the 100s and had the children read some and talk about the number of digits. She asked one of the children to make 745 while she showed the rest of the class 1000s cards. She made several numbers such as 7020 asking the children to read the numbers as she made them. She asked the children if they had seen numbers like this before and after mostly blank looks, pointed to the board. “What year is it?” she asked. 2001 was written as part of the date on the board.

The teacher took the opportunity of the child's understandable but incorrect response and explored mathematical ideas beyond the usual expectations for 5 and 6 year olds but that clearly linked back to their experiences. She was not expecting full understanding at that point, but exploring within the children's experience mathematical concepts that would be revisited in the future. The use of the materials that were appropriately focused for overcoming the misconception also illustrates a teacher who really understands the mathematics that these children need to develop and has a range of materials at her disposal to enable an appropriate choice as the moment arises -An opportunity was taken to move in a mathematically purposeful direction.

In Conclusion

It is interesting to consider the extent to which the list of 25 teacher behaviours and characteristics in Figure 4 has application to other grade bands. We believe that similar research in Grades 4-12 (and possibly beyond), would yield many elements in common with these. Indeed, a discussion of this would be most worthwhile among preservice and inservice teachers.

It was a wonderful privilege to be in the classrooms of dedicated, mathematics education professionals. We have attempted to describe their practice in ways that “ring bells” for readers. We have described classrooms where the enthusiasm, curiosity and strategies of young children are valued and built upon, with lasting effects upon their understanding, their attitudes, their love of mathematics, and their confident views of themselves as learners of mathematics.

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